**atomic sentence:** combination of **names** and **predicates**

**name:** symbol used to refer to some fixed individual object

**predicate symbol:** expresses property of object or **relation** between objects

* aka **relation symbol**
* Likes(max, claire)
  + **predicate:** likes
  + **logical subjects:** ? Max, Claire
  + **referents:** objects referenced by the names max and claire
  + arity: 2
* a=b
  + infix notation
  + Equals(a, b)

**claim:** either true or false; which of these is its **truth value**

* in FOL, atomic sentences are the simplest kinds of claims: just a predicate and its arguments, forming an atomic sentence
* therefore, each atomic sentence in FOL is either true or false

**argument**

* group of statements including one or more premises and one and only one conclusion
* the conclusion is meant to follow from the premises

**statement**

* sentence that is either true or false
* many sentences are not statements

**general musings**

* logic focuses on **validity** of arguments, rather than **soundness**
* all of philosophical logic is meant to provide accounts of the **nature of logical consequence** and the **nature of logical truth**
* given a valid argument, we can look at the actual world and determine if the premises are indeed true; if they are then the conclusion must be true from a logical perspective

**validity** is a property of arguments

* in a valid argument, the conclusion (a statement) follows from (is a logical consequence of) the premises (aka, assumptions, other statements).
* an argument is valid if and only if the conclusion is a logical consequence of the assumptions
* this concept is hypothetical in nature: we can know the argument is valid without knowing anything at all about the truth of the sentences that constitute it
* **special cases**

**logical consequence** is a concept that we apply to one statement relative to other statements

* this concept is hypothetical in nature: we can know that a sentence is a logical consequence without knowing anything at all about its truth value
* in determining logical consequence, it isn’t important whether statements are true or not; what’s important is that hypothetically, if some statements are true then another statement must also be true

**identity relation**

* aka identity predicate
  + Equals(a,b)
  + a = b
* four important principles hold for this relation, and each principle is associated with a formal rule that we can use to make inferences
  + identity elimination
  + reflexivity of identity
  + symmetry of identity (follows from reflexivity and identity elimination)
  + transitivity of identity

**connectives**

* used to connect simple (atomic) claims and form complex claims
* simplest connectives: and, or, not
* these are Boolean operators or Boolean connectives, aka truth-functional connectives

**literal:** sentence that is either atomic or the negation of an atomic

When we speak of **tautology**, **tautological consequence,** or **logical consequence**, we are speaking of a sentence in relation to premises. We are thinking in terms of all the possible truth values of the premises. In the case of the **tautology**, it is really all of the conceivable combinations of truth values of the premises, whether each combination is logically possible or not.

In the case of **logical consequence**, we only consider the combinations that are logically possible.

A sentence is a **tautological consequence** of another set of sentences if whenever each sentence in this latter set is true, the first sentence is true. This means that if a sentence is a tautological consequence of a set of premises, then it is also a logical consequence of that set.

Identity is a predicate. Equals(a,a), aka a = a is an atomic sentence, and has a truth value. It is not logically possible, however, for this predicate to have a truth value of false.

A sentence is a **logical truth** or **logical necessity** if it is true no matter what circumstance we consider. If the sentence is an atomic sentence, it can only have one truth value: true. If it is a complex sentence, then for all logically possible combinations of truth values of the constituent atomic sentences, the complex sentence is true. There may be logically impossible combinations for which the complex sentence is false.

If the complex sentence is true also for the logically impossible combinations, then it is a tautology. Therefore, all tautologies are logical necessities, but not all logical necessities are tautologies. a = a, for example, is not a tautology. We can assign the false truth value to this atomic sentence. This is not logically possible however.

**tautological equivalence, logical equivalence:** these concepts have to do with two statements at the same time. Considering all logically possible combinations of truth values of the two statements, if it is the case that the truth values of the two statements are the same in every one of such combinations, then the two statements are logically equivalent. This means that whenever statement 1 is true, statement 2 must be true, and vice versa. This means that if we take statement 1 as a premise and statement 2 as a conclusion we get a valid argument, and vice-versa. Ie, each statement is provable from the other.

A **tautological equivalence** between two statements occurs when two statements not only have the same truth values in all logically possible circumstances, but also in all logically impossible circumstances. Ie, the two statements have the same truth values in all circumstances that one can build in a truth table.

**if and only if**

shorthand way of saying that two sentences are logically equivalent

* starting with a particular statement, we can sub in logically equivalent pieces (ie we can sub in logical equivalences) to obtain a new statement logically equivalent to the initial one
* we do this to arrive at perhaps a simpler statement that is nonetheless logically equivalent to the first

**there are a few important logical equivalences**

* & distributes over |
  + P & (A | B) ≡ (P & A) | (P & B)
* | distributes over &
  + P | (A & B) ≡ (P | A) & (P | B)
* recall that the main technique for showing that a given conclusion is a logical consequence of some premises is that of a proof
* if a conclusion follows from premises based simply on the structure of the connectives in the premises, then a truth table can also prove this
* there are types of reasoning, ie types of arguments, that rely on other types of things that aren’t simply connective structure

**inference:** steps in reasoning, moving from premises to logical consequences

* we would like to know when, in a proof, we are justified in asserting that a statement is true based on one or more statements that are known to be true.
* that is, how do we know if a statement is a logical consequence of other statements, in cases where a truth table does lot show this?
* there are a few cases that we call rules: particular groups of statements that if present mean that another specific statement follows.

The three simplest of such inference rules are

* conjunction elimination (simplification): we can assert any conjunct individually from a conjunction
* conjunction introduction: given one or more statements known to be true, we can assert the conjunction of all of them
* disjunction introduction: given any statement known to be true, we can assert its disjunction with any other statement

Then we have two other inference rules that are more complex.

* disjunction elimination, aka proof by cases
  + we start with a disjunction we know to be true, and a statement that we would like to prove
  + if there are n disjuncts, we assume each of the n to be true in turn, and prove that S is true
  + since the disjunction is true, then one of the disjuncts is true; since S is a logical consequence of each disjunct, no matter which disjunct is true S will be true.
  + we reach the conclusion that S is a logical consequence of the disjunction
* proof by contradiction
  + we want to prove a sentence S
  + we assume that ~S is true, ie we introduce the negation of S
  + at this point we are assuming that the conjunction of the premises plus ~S is true
  + if from this conjunction, through inference steps, we reach a statement that is false always (ie a contradiction), we have shown that this false statement is logically equivalent to the initial conjunction, so the latter must be false as well. Since we still believe the premises to be true, it must be the case that ~S is false for the conjunction to be false

**thoughts obtained from the internet; slightly different from notes from course**

**on the distinction between statement and proposition**

* for my purposes, it is fine to identify propositions with statements
* an important topic in philosophy is the existence, or lack thereof, of abstract “propositions” independent of the sentences that express them

**proposition**

* statement that the author is proposing for further scrutiny, possibly a proof

**claim**

* proposition that the author claims is true

**statement**

* expresses a proposition
  + the same proposition could instead be expressed by another statement, or multiple statements

**theorem**

* statement that has been proved (or is it a proposition that has been proved?

**corollary**

* theorem that follows in an obvious or simply way from another theorem

**lemma**

* theorem that is very useful in the proof of another theorem or theorems

**Proof without premises**

* shows that its conclusion is a logical truth